MAU22E01 Sheet 2

Use matrix multiplication to find the reflection of the vector $(1,2)$ about the y-axis.

Ex 1 (i) $T(x,y) = (-x, y)$

$$(x', y') = (-x, y)$$

$$\begin{cases} x' = -x \\ y' = +y \end{cases}$$

$$\Rightarrow \begin{aligned} x' &= -1x + 0y \\ y' &= 0x + 1y \end{aligned}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In this case $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\therefore \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} (-1)(1) + (0)(2) \\ (0)(1) + (1)(2) \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Use matrix multiplication to find the orthogonal projection of the vector (1,2,3) to the xy plane.

Ex 1 (i) $T(x, y, z) = (x, y, 0)$

$$\begin{cases} x' = x \\ y' = y \\ z' = 0 \end{cases} \iff \begin{cases} x' = 1x + 0y + 0z \\ y' = 0x + 1y + 0z \\ z' = 0x + 0y + 0z \end{cases}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

In this case $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} (1)(1) + (0)(2) + (0)(3) \\ (0)(1) + (1)(2) + (0)(3) \\ (0)(1) + (0)(2) + (0)(3) \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

Use matrix multiplication to find the image of the vector (1, 2) under rotation through the angle $\frac{-\pi}{4}$ about the origin.

Ex 1(iii) Express x & y in polar coordinates.

$$x = r\cos\phi$$
$$y = r\sin\phi$$

In general, the point can be rotated through an arbitary angle θ counterclockwise about the origin.

$$\Rightarrow \begin{cases} x' = r\cos(\phi+\theta) \\ y' = r\sin(\phi+\theta). \end{cases}$$

$$\Rightarrow \begin{cases} x' = r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ y' = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{cases}$$

$$\Rightarrow \begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \end{cases}$$

$$\Rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In this case $\theta = -\frac{\pi}{4}$ & $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(-\pi/4) & -\sin(-\pi/4) \\ \sin(-\pi/4) & \cos(-\pi/4) \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

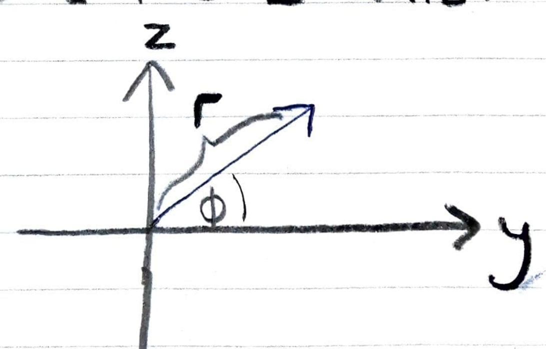
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2(1) + \sqrt{2}/2(2) \\ -\sqrt{2}/2(1) + \sqrt{2}/2(2) \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}$$

Use matrix multiplication to find the image of the vector $(2,-1,1)$ under rotation through an angle $\frac{\pi}{3}$ about x-axis.

Ex 1iv) Express y & z in terms of r & ϕ, where r is the length of the vector in the yz plane and ϕ is the angle the vector makes in the yz plane measured from the y-axis towards the z-axis.



$x = x$
$y = r\cos\phi$
$z = r\sin\phi$

In general, the point can be rotated through an arbitary angle θ about the x-axis (in the yz plane). The angle θ is measured from the y-axis towards the z-axis.

$$\begin{cases} x' = x \\ y' = r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ z' = r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{cases}$$

$$\Rightarrow \begin{cases} x' = x \\ y' = y\cos\theta - z\sin\theta \\ z' = y\sin\theta + z\cos\theta \end{cases}$$

$$\Rightarrow \begin{cases} x' = 1x + 0y + 0z \\ y' = 0x + y\cos\theta - z\sin\theta \\ z' = 0x + y\sin\theta + z\cos\theta \end{cases}$$

$$\Rightarrow \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

In this case $\theta = \frac{\pi}{3}$ & $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi/3) & -\sin(\pi/3) \\ 0 & \sin(\pi/3) & \cos(\pi/3) \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} (1)(2)+(0)(-1)+(0)(1) \\ (0)(2)+(\frac{1}{2})(-1)+(-\frac{\sqrt{3}}{2})(1) \\ (0)(2)+(\frac{\sqrt{3}}{2})(-1)+(\frac{1}{2})(1) \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{-1-\sqrt{3}}{2} \\ \frac{1-\sqrt{3}}{2} \end{pmatrix}$$

For $\bar{u}_1 = (1,-2,-1)$ find parametric equations for the line spanned by the vector.

Ex 2(i) Find the line L spanned by $\bar{u}_1 = (1,-2,-1)$.

L is given parametrically as:

$\bar{x} = k(1,-2,-1) \quad \forall k \in \mathbb{R}$.

$L = \{(k,-2k,-k) : k \in \mathbb{R}\}$.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} k \\ -2k \\ -k \end{pmatrix}$$

$$\Rightarrow \begin{cases} x_1 = k \\ x_2 = -2k \\ x_3 = -k. \end{cases}$$

where $\bar{x} = (x_1, x_2, x_3)$ is a vector used to denote all points on line L.

For $\bar{u}_2 = (1, -2, 0, 0, -1)$ find parametric equations for the line spanned by the vector.

Ex 2i) Find the line L spanned by $\bar{u}_2 = (1, -2, 0, 0, -1)$.

L is given parametrically as:

$$\bar{x} = k(1, -2, 0, 0, -1) \quad \forall\, k \in \mathbb{R}.$$

$$L = \{(k, -2k, 0, 0, -k) : k \in \mathbb{R}\}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} k \\ -2k \\ 0 \\ 0 \\ -k \end{pmatrix}$$

$$\Rightarrow \begin{cases} x_1 = k \\ x_2 = -2k \\ x_3 = 0 \\ x_4 = 0 \\ x_5 = -k \end{cases}$$

where $\bar{x}$ is a vector used to denote all points on line L.

Ex 2ii) Find implicit equations that determine the line spanned by $\bar{u}_1 = (1, -2, -1)$.

$$\begin{cases} x_1 = k \\ x_2 = -2k \\ x_3 = -k \end{cases}$$

$\Rightarrow \quad k = x_1$

The implicit equations are :

$$\therefore \begin{cases} x_2 = -2x_1 \\ x_3 = -x_1 \end{cases}$$

Ex 2ii) Find implicit equations that determine the line spanned by $\bar{u}_2 = (1, -2, 0, 0, -1)$

$x_1 = k$
$x_2 = -2k$
$x_3 = 0$
$x_4 = 0$
$x_5 = -k$

$\Rightarrow \quad k = x_1$

The implicit equations are :

$$\therefore \begin{cases} x_2 = -2x_1 \\ x_3 = 0 \\ x_4 = 0 \\ x_5 = -x_1 \end{cases}$$